University of Tuebingen

Laboratory internship II

TE

Thermal Emission

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1 Experimental setup & procedure

The electron emitting cathode, as well as the andode are wired according to figure 1. We apply a voltage of $U_G = 3$V (contrary to the 2V in the fiure). Following the figure, we read off the voltage and the current three times. Firstly, we measure the heating current I_H, which provides the tube with the heat required in order for electrons to escape the cathode.

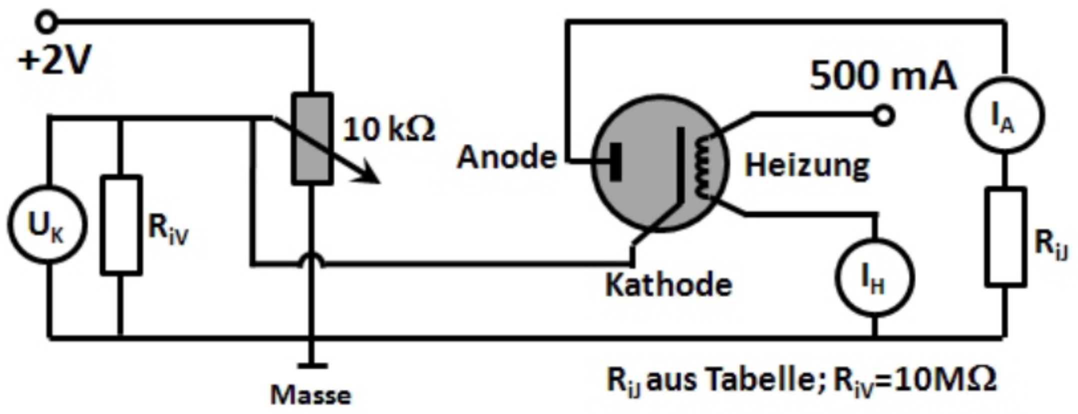

Figure 1: Experimental setup — *Source: Thermische Emission / PhysikerV20210104*[1]

Secondly, we measure the current that passes through the anode I_A. Finally, the cathode temperature T_K is calcualted.
The heating current I_H varies between 450mA, 500mA and 550mA, whereas it is left constant over the three different measurements. In each section we vary the voltage U_G from 0V in approximately 20 equally spaced steps until we arrive at $I_A < 1.5$nA and record the corresponding values for I_A.

[1]http://pisrv1.am14.uni-tuebingen.de/praktikum/anfaenger/TE.pdf — Accessed 04/26/2021 11:39

2 Theoretical background

A beam of electron is generated by a diode. Electrons are removed from the cathode and travel towards the anode due to the electric field that is present bewtween them. In order for the electrons to actually arrive at the anode, they need to have a sufficiently high energy. That is, they not only need to cope the potential of the atoms in the material, but also the voltage

$$U_{AK} = \phi_K - \phi_A = -U_G \tag{1}$$

between the cathode and the anode. Thus, one needs to have a current that is equal to or larger than

$$I_A(U_G, T) = I_S(T)e^{\frac{eU_{AK}}{k_B T}} = I_S(T)e^{-\frac{eU_G}{k_B T}}, \tag{2}$$

where $I_S(T) = I_A(0, T)$ is the saturation current, i.e. the maximum current for a given temperature. This holds for the regime $U_G < 0$ in figure 2. Only electrons with a sufficiently high energy reach the anode. SOMMERFELD and NORDHEIM found that for homogenous surfaces with temperature dependent work it holds

$$I_S(T) = A_0 F T^2 e^{-\frac{W_K}{k_B T}}, \tag{3}$$

where A_0 denotes a constant and F stands for the area of the cathode. Furthermore, the work required to remove an electron from the cathode is defined as W_K. The relation above usually only holds for homogenous surfaces with temperature dependent work. In our setup this is approximately fulfilled for the anode with a tube that has a ratio of $\frac{R}{r} \approx 1.5$, where R denotes the radius of the anode and r the one of the cathode. For the cathode, the condition above does not hold, but they rather fulfill a similar relation, called the RICHARDSON equation:

$$I_{S,A}(T) = A_R F T^2 e^{-\frac{W_K}{k_B T}}. \tag{4}$$

The reason why the latter equation describes the cathode is that it holds for the saturation of the cathode current (see figure 2), that is, for the case that all electrons emitted from the cathode reach the anode. The saturation current then does not increase anymore if U_G is further increased. In this regi,e only with an increase of the cathode temperature the current I_S can be increased.

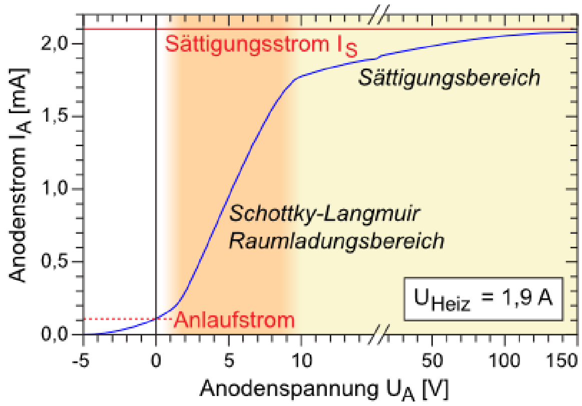

Figure 2: Diode emission line — *Source: LP Goettingen* [2]

Therefore, we observe the important proportionality

$$I_S(T) \propto T^2. \tag{5}$$

[2]https://lp.uni-goettingen.de/get/text/4256 — Accessed 04/26/2021/12:53

The Schottky effect, which describes the decrease of work needed to remove an electron from a material due to an electric field, leads to another constant $-\Delta W$ in the exponetnials of the previous equations.

Finally, the different values for the work required to remove an electron from the cathode and the anode has to be considered. This results in a correction to equation 1, which given as

$$U_{AK}^{eff} = -U_G + \frac{1}{e}(W_K - W_A). \tag{6}$$

Here, W_K and W_A is the work required to remove an electron from the cathode and the anode respectively.
Consiering this new voltage and by taking the expression 3 for I_S into account, we obtain

$$I_A(U_G, T) = A_0 F T^2 e^{-\frac{W_A + eU_G}{k_B T}}, \tag{7}$$

which is the correct version of equation 2 (Schkotty effect neglected).
Assuming a sufficiently weak dependence of W_A on T, we obtain by rearranging the previous equation

$$W_A(T) = k_B T \ln\left(\frac{A_0 F T^2}{I_A(0,T)}\right). \tag{8}$$

Taking into account the Schkotty effect, the current gets a correction factor

$$I_A(U_G, T) = A_0 F T^2 e^{-\frac{W_A + eU_G - \Delta W}{k_B T}}. \tag{9}$$

Last, but not least, let us derive an analytical expression for ΔW.
The Coulomb force in front of a surface at distance x can be described with a mirror charge (opposite charge) at $-x$, leading to the expression

$$F = -\frac{e^2}{4\pi\epsilon_0 (2x)^2}, \tag{10}$$

from which we obtain the potential

$$W(x) = W_0 - \int_{\text{inf}}^{x} \frac{e^2 dx'}{4\pi\epsilon_0 (2x')^2} = W_0 - \frac{e^2}{16\pi\epsilon_0 x}. \tag{11}$$

By adding the force of the field, namely $F = eE$, we obtain

$$W(x) = W_0 - \frac{e^2}{16\pi\epsilon_0 x} + eEx. \tag{12}$$

Taking derivatives with respect to x and finding the maximum yields the maximum work required to push out an electron:

$$x_{max} = \sqrt{\frac{e}{16\pi\epsilon_0 E}} \tag{13}$$

$$\rightarrow W_{max} = W_0 - \sqrt{\frac{e^3 E}{4\pi\epsilon_0}}. \tag{14}$$

Plugging this into the formula for I_A, we obtain

$$I_A(U_G, T) = A_0 F T^2 e^{-\frac{W_A + eU_G - \sqrt{\frac{e^3 E}{4\pi\epsilon_0}}}{k_B T}}. \tag{15}$$

3 Results & analysis

3.1 Task 1

By taking the natural logarithm on both sides of equation 9, we obtain

$$\ln(I_A) = -\frac{e}{k_B T} U_G + \ln\left(A_0 F T^2\right) - \frac{W_A}{k_B T} = m U_G + b, \tag{16}$$

where m denotes the slope of the linear fit and b denotes the displacement on the y axis.
Using

$$\ln(I_A(U_G)) = m U_G + b \tag{17}$$

$$\rightarrow T = \frac{-e}{k_B m}, \tag{18}$$

linear regression yields

$I_A(U_G)[mA]$	m	b	T [K]
444	-11.74±0.125	-4.47±0.12	998.45 ±10.53
496	-11.22 ±0.19	-2.60 ±0.21	1.034 ±17.72
545	-10.48 ±0.27	-1.42 ±0.346	1107.2±28.83

Table 1: Calcuated temperatures

where Gaussian error methods have been used to determine the error for the temperature T. The linear trend lines can be seen in the following figure for each current.

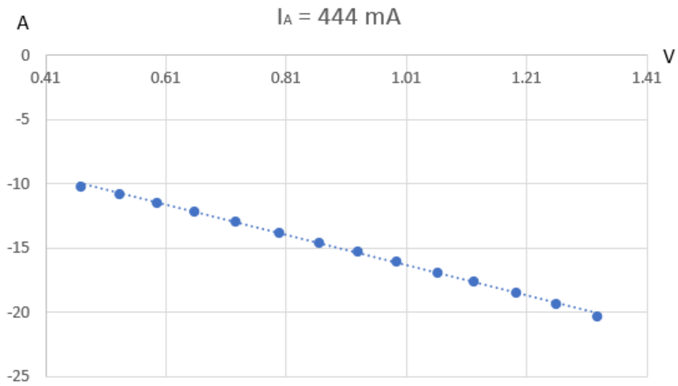

Figure 3: Current $I = 444$mA

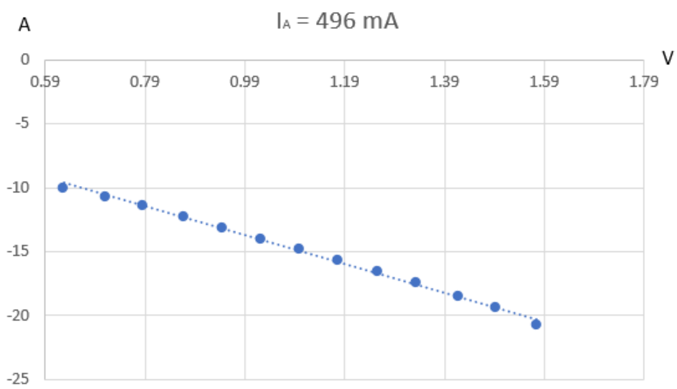

Figure 4: Current $I = 496$mA

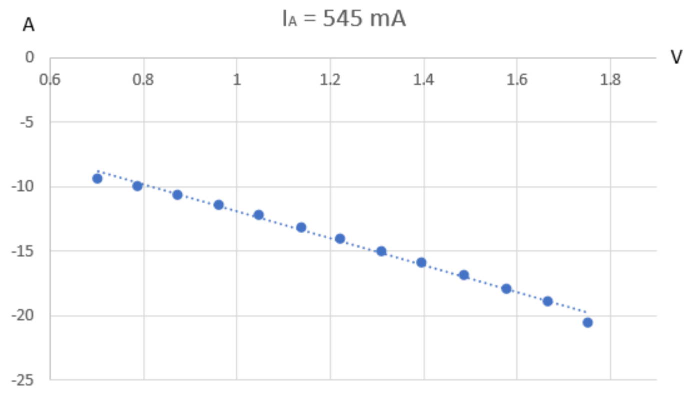

Figure 5: Current $I = 545\text{mA}$

Note the logarithmic scaling on the y axis of the figures above.
As expected, the linear relation can be observed. Furthermore, we obtain an increasing temperature (see table 5) as expected from the theory.

3.2 Task 2

The task is to determine the anode work function W_A for or tree thermionic diodes With the formula number 8 from the theoretical Backround we could determine W_A

$$W_A(T) = k_B T \ln\left(\frac{A_0 F T^2}{I_A(0,T)}\right)$$

Whit dthe temperatures from task 1 and the current $I_A(0,T)$ when the electrical tension is zero. So we recibe the following solutions

$\text{I}_A(U_G)[mA]$	$\text{W}_A[eV]$	$\Delta W_A[ev]$
444	1.72	± 0.02
496	1.73	± 0.033
545	1.82	± 0.034

Table 2: Calcuated work function W_A

3.3 Task 3

For the Schottky effect see 2, where we go through the theory including the Shottky effect.

3.4 Task 4

The Task is to determine the emission efficiency η of two different hot cathodes. Both cathodes have to deliver a current density of $j = 0.5\frac{A}{cm^2}$. The definition for teh emission efficiency is given by the following realation:

$$\eta = \frac{I_s}{P_H}$$

I_s describe the emission Current and P_H the heating power. With the formula of Richardson:

$$I_s(T) = A_R F T^2 e^{-\frac{W_K}{k_B T}}$$

And the Law of Boltzmann:

$$P_H = \varepsilon \sigma F T^4$$

we recibe the follwing relation for the emission efficiency:

$$\eta = \frac{A_R}{\varepsilon\sigma T^2} e^{-\frac{W_K}{k_B T}}$$

To dicate the efficiency of our tungsten athode ($A_R = 60\frac{A}{cm^2K^2}$ and $W_K = 4.35eV$) and our oxide cathode ($A_R = 0.046\frac{A}{cm^2K^2}$ and $W_K = 1.2eV$), we have to know the Temperatur of The both cathodes, in case a current density of $j = 0.5\frac{A}{cm^2}$ Together with the formula of the current density:

$$j_s = A_R T^2 e^{-\frac{W_K}{k_B T}}$$

and the other formulas, we get an equation to determine the Temperature we need for the efficiency:

$$0 = 2\ln T - \frac{W_K}{k_B T} + \ln A_R - \ln j_s$$

This equation can‘t be solved analytical. Numerical we recibe two temperatures for the two different mediums.

$$T_{Wolfram} = 2565.85\ K$$
$$T_{Oxid} = 1183.4\ K$$

Whit this temperatures we could determine the emission efficiency

$$\eta_{Wolfram} = 0.007\ \frac{A}{W}$$
$$\eta_{Oxid} = 0.2366\ \frac{A}{W}$$

The results coincide in the order of magnitude with the literature values of $T_{Wolfram} \approx 2700K$ at normal pressure.